Calcolare $\int\int_A f[x, y]\, dx\, dy$ **con A e f[x, y] come indicato**

1) $f[x, y] = \dfrac{x^2}{\left(x^2 + y^2\right)^2}$,

$A = \left\{(x, y);\ 1 \le x^2 + y^2 \le 4\right\}$

Indicazioni per la risoluzione :

```
Simplify[{ρ * f[ρ Cos[θ], ρ Sin[θ]]}]
```

$$\left\{\frac{\text{Cos}[\theta]^2}{\rho}\right\}$$

```
Simplify[{∫_0^(2 π) Cos[θ]^2/ρ dθ, ∫_1^2 ∫_0^(2 π) Cos[θ]^2/ρ dθ dρ}]
```

$$\left\{\frac{\pi}{\rho}, \pi\,\text{Log}[2]\right\}$$

2) $f[x, y] = \dfrac{x^2 y}{x + 3}$,

$A = \left\{(x, y);\ x^2 + y^2 \le 16,\ x \le 0,\ y \ge 0\right\}$

Indicazioni per la risoluzione :

```
Simplify[{∫_0^Sqrt[16-x^2] x^2 y/(x - 3) dy, ∫_-4^0 ∫_0^Sqrt[16-x^2] x^2 y/(x - 3) dy dx}]
```

$$\left\{\frac{x^2\left(8 - \frac{x^2}{2}\right)}{-3 + x}, \frac{7}{2}\left(4 + 9\,\text{Log}[3] - 9\,\text{Log}[7]\right)\right\}$$

3) $f[x, y] = \dfrac{x}{y}$,

$A = \left\{(x, y);\ x^2 + (y - 2)^2 \le 1,\ x \ge 0\right\}$

Indicazioni per la risoluzione :

```
Simplify[{ρ * f[ρ Cos[θ], 2 + ρ Sin[θ]]}]
```

$$\left\{\frac{\rho^2\,\text{Cos}[\theta]}{2 + \rho\,\text{Sin}[\theta]}\right\}$$

```
Simplify[{∫_(-π/2)^(π/2) ρ^2 Cos[θ]/(2 + ρ Sin[θ]) dθ, ∫_0^1 ∫_(-π/2)^(π/2) ρ^2 Cos[θ]/(2 + ρ Sin[θ]) dθ dρ}]
```

$$\left\{\rho\left(-\text{Log}[2 - \rho] + \text{Log}[2 + \rho]\right), 2 - \frac{3\,\text{Log}[3]}{2}\right\}$$

Indicazioni per la risoluzione (altro modo) :

```
Simplify[{∫_0^Sqrt[-y^2+4 y-3] x/y dx, ∫_1^3 ∫_0^Sqrt[-y^2+4 y-3] x/y dx dy}]
```

$$\left\{2 - \frac{3}{2\,y} - \frac{y}{2},\ 2 - \frac{3\,\text{Log}[3]}{2}\right\}$$

4) $f[x, y] = \dfrac{x+y}{x-y+4}$,

$A = \{(x, y);\ x^2 + y^2 \le 4\}$

Indicazioni per la risoluzione :

```
Simplify[{ρ * f[ρ Cos[θ], ρ Sin[θ]]}]
```

$$\left\{\frac{\rho^2\,(\text{Cos}[\theta] + \text{Sin}[\theta])}{4 + \rho\,\text{Cos}[\theta] - \rho\,\text{Sin}[\theta]}\right\}$$

$$\int \frac{\rho^2\,(\text{Cos}[\theta] + \text{Sin}[\theta])}{4 + \rho\,\text{Cos}[\theta] - \rho\,\text{Sin}[\theta]}\,d\theta$$

$$-\rho\,\text{Log}[4 + \rho\,\text{Cos}[\theta] - \rho\,\text{Sin}[\theta]]$$

```
Simplify[{∫_0^(π/4) ρ^2 (Cos[θ] + Sin[θ])/(4 + ρ Cos[θ] - ρ Sin[θ]) dθ, ∫_0^2 ∫_0^(π/4) ρ^2 (Cos[θ] + Sin[θ])/(4 + ρ Cos[θ] - ρ Sin[θ]) dθ dρ}]
```

$$\left\{\rho\,\text{Log}\left[\frac{4+\rho}{4}\right],\ 3 - \text{Log}\left[\frac{729}{64}\right]\right\}$$

5) $f[x, y] = e^{-x^2-y^2}$,

$A = \{(x, y);\ x^2 + y^2 \le 4,\ y \ge 0\}$

Indicazioni per la risoluzione :

```
Simplify[{ρ * f[ρ Cos[θ], ρ Sin[θ]]}]
```

$$\left\{e^{-\rho^2}\,\rho\right\}$$

```
Simplify[{∫_0^π e^(-ρ^2) ρ dθ, ∫_0^2 ∫_0^π e^(-ρ^2) ρ dθ dρ}]
```

$$\left\{e^{-\rho^2}\,\pi\,\rho,\ \frac{\left(-1 + e^4\right)\,\pi}{2\,e^4}\right\}$$

6) $f[x, y] = \dfrac{x}{\left(x^2+y^2\right)^2}$,

$A = \{(x, y);\ x^2 + y^2 \le 4,\ x \ge 1\}$

Indicazioni per la risoluzione :

```
Simplify[{∫_(1/Cos[θ])^2 Cos[θ]/ρ^2 dρ, ∫_(-π/3)^(π/3) ∫_(1/Cos[θ])^2 Cos[θ]/ρ^2 dρ dθ}]
```

$$\left\{\left(-\frac{1}{2} + \text{Cos}[\theta]\right)\,\text{Cos}[\theta],\ -\frac{\sqrt{3}}{4} + \frac{\pi}{3}\right\}$$

7) $f[x, y] = \dfrac{x+1}{3-y}$,

$A = \{(x, y);\ (x+1)^2 + (y-1)^2 \le 1,\ x \le -1,\ y \ge 1\}$

Indicazioni per la risoluzione :

```
Simplify[{ρ * f[-1 + ρ Cos[θ], 1 + ρ Sin[θ]]}]
```

$$\left\{\frac{\rho^2 \cos[\theta]}{2 - \rho \sin[\theta]}\right\}$$

```
Simplify[{∫_(π/2)^(3π/2) (ρ^2 Cos[θ])/(-2 + ρ Sin[θ]) dθ, ∫_0^1 ∫_(π/2)^(3π/2) (ρ^2 Cos[θ])/(2 - ρ Sin[θ]) dθ dρ}]
```

$$\left\{\rho\,(\text{Log}[-2-\rho] - \text{Log}[-2+\rho]),\ -2 + \frac{3\,\text{Log}[3]}{2}\right\}$$

8) $f[x, y] = \dfrac{x}{x^2 + y^2 + 2}$,

$A = \left\{(x, y);\ x^2 + y^2 \le 2,\ x - y \le 0\right\}$

Indicazioni per la risoluzione :

```
Simplify[{ρ * f[ρ Cos[θ], ρ Sin[θ]]}]
```

$$\left\{\frac{\rho^2 \cos[\theta]}{2 + \rho^2}\right\}$$

```
Simplify[{∫_(π/4)^(5π/4) (ρ^2 Cos[θ])/(2 + ρ^2) dθ, ∫_0^Sqrt[2] ∫_(π/4)^(5π/4) (ρ^2 Cos[θ])/(2 + ρ^2) dθ dρ}]
```

$$\left\{-\frac{\sqrt{2}\ \rho^2}{2 + \rho^2},\ \frac{1}{2}\ (-4 + \pi)\right\}$$

9) $f[x, y] = y$,

$A = \left\{(x, y);\ x^2 + y^2 + 6\,y \le 0\right\}$

Indicazioni per la risoluzione :

```
Simplify[{ρ * f[ρ Cos[θ], -3 + ρ Sin[θ]]}]
```

```
{ρ (-3 + ρ Sin[θ])}
```

```
Simplify[{∫_0^(2π) ρ (-3 + ρ Sin[θ]) dθ, ∫_0^3 ∫_0^(2π) ρ (-3 + ρ Sin[θ]) dθ dρ}]
```

```
{-6 π ρ, -27 π}
```

10) $f[x, y] = \dfrac{1}{x^2}$,

$A = \left\{(x, y);\ 1 \le x^2 + y^2 \le 4,\ \dfrac{\sqrt{3}}{3}\,x \le y \le x\right\}$

Indicazioni per la risoluzione :

```
Simplify[{ρ * f[ρ Cos[θ], ρ Sin[θ]]}]
```

$$\left\{\frac{\text{Sec}[\theta]^2}{\rho}\right\}$$

```
Simplify[{∫_(π/6)^(π/4) Sec[θ]^2/ρ dθ, ∫_1^2 ∫_(π/6)^(π/4) Sec[θ]^2/ρ dθ dρ}]
```

$$\left\{-\frac{-3+\sqrt{3}}{3\,\rho},\ \left(1-\frac{1}{\sqrt{3}}\right)\text{Log}[2]\right\}$$

11) f[x, y] = $\frac{y}{x}$,

A = $\{(x, y);\ x^2+y^2 \le 4,\ x^2+y^2-2y \ge 0,\ 0 \le x \le y\}$

Indicazioni per la risoluzione :

```
Simplify[{ρ * f[ρ Cos[θ], ρ Sin[θ]]}]
```

$$\{\rho\,\text{Tan}[\theta]\}$$

```
Simplify[{∫_(2 Sin[θ])^2 ρ Tan[θ] dρ, ∫_(π/4)^(π/2) ∫_(2 Sin[θ])^2 ρ Tan[θ] dρ dθ}]
```

$$\left\{\text{Sin}[2\theta],\ \frac{1}{2}\right\}$$

12) f[x, y] = $\frac{x^2 y}{x^2+y^2}$,

A = $\{(x, y);\ 4 \le x^2+y^2 \le 9,\ y \ge -|x|\}$

Indicazioni per la risoluzione :

```
Simplify[{ρ * f[ρ Cos[θ], ρ Sin[θ]]}]
```

$$\{\rho^2\,\text{Cos}[\theta]^2\,\text{Sin}[\theta]\}$$

```
Simplify[{∫_(-π/4)^(5π/4) ρ^2 Cos[θ]^2 Sin[θ] dθ, ∫_2^3 ∫_(-π/4)^(5π/4) ρ^2 Cos[θ]^2 Sin[θ] dθ dρ}]
```

$$\left\{\frac{\rho^2}{3\sqrt{2}},\ \frac{19}{9\sqrt{2}}\right\}$$

13) f[x, y] = $x y^2$,

A = $\{(x, y);\ x^2+y^2 \le 4,\ x+y \ge 0\}$

Indicazioni per la risoluzione :

```
Simplify[{ρ * f[ρ Cos[θ], ρ Sin[θ]]}]
```

$$\{\rho^4\,\text{Cos}[\theta]\,\text{Sin}[\theta]^2\}$$

```
Simplify[{∫_(-π/4)^(3π/4) ρ^4 Cos[θ] Sin[θ]^2 dθ, ∫_0^2 ∫_(-π/4)^(3π/4) ρ^4 Cos[θ] Sin[θ]^2 dθ dρ}]
```

$$\left\{\frac{\rho^4}{3\sqrt{2}},\ \frac{16\sqrt{2}}{15}\right\}$$